Questions and answers for Module 2

R. Ganapathy
Senior Assistant Professor
School Of Electrical and Electronics Engineering
SASTRA University

Table of Content

1 Questions

1. What are the properties of a linear vector space ?
2. What is the dimensionality of a linear vector space ?
3. When is a quantum mechanical operator said to be linear ?
4. What are the properties of a Hermitian operator ?
5. State the first postulate of quantum mechanics.

2 Answers

1. The various properties of a linear vector space are as follows:

   - $|V_i\rangle + |V_j\rangle = |V_j\rangle + |V_i\rangle$, known as commutative property of addition,
   - $\left(|V_i\rangle + |V_j\rangle\right) + |V_k\rangle = |V_i\rangle + \left(|V_j\rangle + |V_k\rangle\right)$, known as associative property of addition,
   - existence of a unique null vector $|\emptyset\rangle$ in V such that $|V_i\rangle + |\emptyset\rangle = |V_i\rangle = |\emptyset\rangle + |V_i\rangle$, thereby existing as an identity element of addition,
   - existence of a unique inverse $|-V_i\rangle$ in addition such that $|V_i\rangle + |-V_i\rangle = |\emptyset\rangle$,
   - $\alpha\left(|V_i\rangle + |V_j\rangle\right) = \alpha|V_i\rangle + \alpha|V_j\rangle$, pertaining to scalar multiplication,
   - $(\alpha+\beta)|V_i\rangle = \alpha|V_i\rangle + \beta|V_i\rangle$, also pertaining to scalar multiplication and
   - $\alpha(\beta|V_i\rangle) = (\alpha\beta)|V_i\rangle$, also pertaining to scalar multiplication.

2. The dimensionality of a linear vector space or linear vector space is decided by the maximum number of linear independent vectors in that linear vector space. Thus if there are at most N number of linear independent vectors, the linear vector space is N dimensional.

3. An operator, say, $\hat{\Phi}$ is said to be linear, if it satisfies the following mathematical relations:

   - $\hat{\Phi}\beta|V\rangle = \beta\hat{\Phi}|V\rangle$.
   - $\langle V|\hat{\Phi}\beta = \langle V|\beta\hat{\Phi}$.
   - $\hat{\Phi}(\alpha|V_1\rangle + \beta|V_2\rangle) = \alpha\hat{\Phi}|V_1\rangle + \beta\hat{\Phi}|V_2\rangle$.
   - $(\langle V_1|\alpha + \langle V_2|\beta)\hat{\Phi} = \langle V_1|\hat{\Phi}\alpha + \langle V_2|\hat{\Phi}\beta$.

4. For a Hermitian operator, all the eigen values are real. For distinct eigen values, the corresponding eigen vectors are orthogonal to each other.

5. The state of a quantum mechanical particle is represented by $|\psi\rangle$ in a Hilbert space which can be defined as a linear vector space consisting of an inner product space.